

# Working Group on Nonsmooth Differential Geometry and Its Applications

## Introduction

This working group aims to study stability problems under lower Ricci curvature bounds. Our focus will be on metric measure spaces and analysis on them. The main goal is to understand analytic and geometric tools that allow one to formulate and prove stability and convergence results in nonsmooth settings.

We will start with a systematic introduction to optimal transport theory following Ambrosio, Brué, and Semola's monograph. Afterwards, we will study Gigli's article on stability theory in the presence of synthetic lower Ricci curvature bounds.

## Tentative Program

1. **Metric measure spaces:** definitions and different types of convergences (Reference [3]).
2. **Optimal transport I:** basic definitions, existence results, Kantorovich–Rubinstein duality (Reference [1]).
3. **Optimal transport II:** geometry of the space of measures, Wasserstein distances, analysis on metric spaces, and the dynamical formulation of optimal transport (Reference [1]).
4. **Optimal transport III:** Wasserstein geodesics, non-branching, curvature and gradient flows (Reference [1]).
5. **Optimal transport IV:** heat flow, optimal transport, and Ricci curvature (Reference [1]).
6. **Curvature–dimension conditions (CD):** definition, intuition, and importance (Reference [3]).
7. **First-order analysis:** first-order Sobolev functions, the heat flow as a gradient flow (Reference [3]).
8. **RCD condition:** (Reference [3]).
9. **Differential calculus in a nonsmooth setting:** (Reference [2]; [3]).
10. **RCD condition and heat flow:** (Reference [2]; [3]).
11. **Lower semicontinuity and convergence results:** (Reference [3]).
12. **Mosco convergence for the Laplacian and convergence of the Willmore energy:** (Reference [3]).

## Main References

### References

- [1] L. Ambrosio, E. Brué, D. Semola, *Lectures on Optimal Transport*, Springer, Cham, 2024. DOI: <https://doi.org/10.1007/978-3-031-76834-7>.
- [2] N. Gigli, E. Pasqualetto, *Lectures on Nonsmooth Differential Geometry*, Springer, Cham, 2020. DOI: <https://doi.org/10.1007/978-3-030-38613-9>.
- [3] N. Gigli, *De Giorgi and Gromov Working Together: A survey about stability results in connection with lower Ricci bounds*, preprint, 2023, <https://arxiv.org/abs/2306.14604v1>.

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