p-adic Langlands Program for GL_2

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1 Introduction

Langlands program has become a very popular subject nowadays. Roughly speaking, it aims to build a bridge between automorphic representations and Galois representations. As its p-adic local analogue, the p-adic Langlands program studies the relation between n-dimensional $GL_n(F)$-representations over E and n-dimensional G_F-representations over E, where F is a finite extension of $\mathbb{Q}_p$, E is a finite extension of $\mathbb{Q}_l$, with both p and l prime numbers, and G_F is the absolute Galois group of F. In general case, we do not know too much. What we understand most clearly is the case when $n = 2$, $F = \mathbb{Q}_p$, $p = l$, the so-called GL_2-case. In this case, by the work of Breuil, Colmez, Emerton, Paškūnas...we totally understand the correspondence between 2-dimensional $GL_2(\mathbb{Q}_p)$-representations and 2-dimensional $G_{\mathbb{Q}_p}$-representations.
In this workshop, we will fully understand (hopefully) how to build this correspondence, by understanding the behavior of p-adic representations and the relationships between Galois representations and (φ, Γ)-modules. We will follow three volumes' papers—Astérisque 319, 330 and 331, mostly focused on the previous two volumes and also other classic references.

2 Contact

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3 Schedule

Time and place: On each Tuesday's afternoon 16:40-18:40 at Jussieu 15-16-101.

- 1: Introduction
- 2: Introduce filtered (φ, N, G_K)-modules and (φ, Γ_K)-modules; prove certain category equivalence between them, following [Ber]
- 3: Introduce Fontaine's rings and prove an analogue of Dieudonné-Manin theorem, following [Col1] section 1-4
- 4: Calculate the Galois cohomology on Fontaine's rings, following [Col1] section 5-10
- 5: Introduce the Artin conductor and valuation convergence filtration on B_{dR}^+; prove theorem 0.6 in [Col2]
- 6: Classify all 2-dimensional trianguline representation, following [Col3]
- 7: Prove the slope filtration theorem, following [Ked]
- 8: Introduce Sen's methods and its application on p-adic representations, following [BC]
- 9: Do p-adic analysis, following [Col4]
- 10: Introduce the functor $D \mapsto D^\sharp \boxtimes \mathbb{Q}_p$ and $D \mapsto D^\natural \boxtimes \mathbb{Q}_p$, following [Col5] section 2-3

- 11: Finish the remaining part of [Col5]
- 12: Introduce and study the Banach space $B(V)$ attached to a 2-dimensional potentially cristalline irreducible $G_{\mathbb{Q}_p}$-representaiton V, following [BB]
- 13: Study the p-adic Langlands correspondence at the unitary principal series case, following [Col6]
- 14: Study the p-adic Langlands correspondence, following [Col7]

4 Reference

[Ber]L.Berger-Équations différentielles p-adiques et (φ, N)-modules filtrés
[BB]L.Berger, C.Breuil-Sur quelques représentations potentiellement cristallines de $GL_2(\mathbb{Q}_p)$
[BC]L.Berger, P.Colmez-Familles de représentations de de Rham et monodromie p-adique
[Col1]P.Colmez-Espaces vectoriels de dimension finie et représentations de de Rham
[Col2]P.Colmez-Conducteur d'Artin d'une représentation de de Rham
[Col3]P.Colmez-Représentations triangulines de dimension 2
[Col4]P.Colmez-Fonctions d'une variable p-adique
[Col5]P.Colmez-(φ, Γ)-modules représentations du mirabolique de $GL_2(\mathbb{Q}_p)$
[Col6]P.Colmez-La série principale unitaire de $GL_2(\mathbb{Q}_p)$
[Col7]P.Colmez-Représentations de $GL_2(\mathbb{Q}_p)$ et (φ, Γ)-modules
[Ked]K.S.Kedlaya-Slope filtrations for relative Frobenius