Working Group: Groups and curvature

October 2025

Introduction

Geometric group theory is founded on the observation that the algebraic properties of a discrete group are closely related to the geometry of the spaces it acts on. The goal of this working group is to explore a few basic results of this subject by describing links between growth, curvature and hyperbolicity. It will be separated in two parts. First we will study algebraic properties of the fundamental groups of a Riemannian manifold which are related to its curvature. Then we will turn to hyperbolic groups, a generalization of groups acting on a negatively-curved space.

Curvature and fundamental groups

Let M be a Riemannian manifold, $G = \pi_1(M)$ its fundamental group and K its sectional curvature. Some results that we will try to prove during this working group are the following.

- Bonnet's Theorem : if there exists $\kappa > 0$ such that $K \geq \kappa^2$, then $\mathrm{diam}(M) < \pi/\kappa$ and G is finite.
- Cartan-Hadamard : if $K \leq 0$, then G contains no non-trivial element of finite order.
- Preissmann's Theorem : if $K < 0$, then any abelian subgroup of G is either trivial or isomorphic to $\mathbb{Z}$.
- If M is compact and has non-negative mean curvature, then G has polynomial growth.
- Milnor's Theorem : if $K < 0$, then G has exponential growth. Possible complement : if M is the genus g surface with $g \geq 2$, then G has uniformly exponential growth.

Hyperbolic groups

- definitions and quasi-isometry

- boundary of hyperbolic groups. Possible application : in hyperbolic groups, the centralizer of an infinite order element is virtually infinite cyclic
- characterization of finitely generated hyperbolic groups by linearity of the Dehn function
- for hyperbolic groups, being residually finite is equivalent to being virtually torsion free

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