

Hyperkähler geometry working group

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1 Introduction

The goal of this working group is to understand the geometric properties of hyperkähler manifolds to study irreducible holomorphic symplectic manifolds via the "period map". The period map is an important tool that allows us, for example, to compare K3 surfaces by looking only at their H^2 ; this is the content of the global Torelli theorem.

As the Hodge theory course will begin in November, we will first focus on the holonomy group of a Riemannian manifold which was historically the starting point of the theory of hyperkähler manifolds through Berger's classification. After that we will turn to Calabi-Yau manifolds and finally in hyperkähler ones.

We will mainly follow the structure of [Mar03], but [Deb20] may also be useful. Claire Voisin has also given some lectures at the Collège de France [Voi17].

2 Program

- 1) Holonomy groups and Berger's classification
- 2) Calabi conjecture and Yau's Proof
- 3) Variation of Hodge Structure
- 4) Holomorphic symplectic manifolds and moduli spaces

3 Contact

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References

- [Deb20] Olivier Debarre. "Hyperkähler manifolds". In: (2020). URL: <https://arxiv.org/abs/1810.02087>.
- [Mar03] Daniel Huybrechts Mark Gross Dominic Joyce. *Calabi-Yau Manifolds and Related Geometries*. Springer, 2003.
- [Voi17] Claire Voisin. *Variétés hyper-kählériennes*. Lecture at Collège de France. Collège de France. 2017. URL: https://youtu.be/aSCom3dWpDw?si=Ao12HtIF5i5WNb_-.